

**LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034****B.Sc. DEGREE EXAMINATION – PHYSICS****FOURTH SEMESTER – APRIL 2023****UST 4401 – PROBABILITY AND DISTRIBUTIONS**

Date: 04-05-2023

Dept. No. 

Max. : 100 Marks

Time: 09:00 AM - 12:00 NOON

**SECTION A - K1 (CO1)****Answer ALL the Questions****(10 x 1 = 10)****1. Define the following**

- a) Probability of an event.  
 b) Marginal distribution.  
 c) Discrete Uniform distribution.  
 d) Continuous distributions.  
 e) Random walk.

**2. Fill in the blanks**

- a) If A and B are independent, then  $P(A \cap B) =$  \_\_\_\_\_.  
 b) The first raw moment is called \_\_\_\_\_ of the distribution.  
 c) Variance > mean in \_\_\_\_\_ distribution.  
 d) The mean of gamma distribution with one parameter is \_\_\_\_\_.  
 e) A discrete parameter Markov process is called \_\_\_\_\_.

**SECTION A - K2 (CO1)****Answer ALL the Questions  
10)****(10 x 1 =****3. Match the following**

- |  |                    |
|--|--------------------|
| a) At least one of the events A or B occurs  | Poisson            |
| b) If X and Y are independent then $E(XY) =$ | $N(\mu, \sigma^2)$ |
| c) Mean=Variance                             | Markov process     |
| d) Normal distribution                       | AUB                |
| e) Weather prediction models                 | $E(X)E(Y)$         |

**4. True or False**

- a) The probability of impossible event is 1.  
 b) Tossing a coin is an example of a random experiment.  
 c) When the population size increases hypergeometric distribution tends to binomial distribution.  
 d) If X and Y are two independent normal variates, then X-Y is not a normal variate.  
 e) In Markov process future depends on both present and past states.

**SECTION B - K3 (CO2)****Answer any TWO of the following  
20)****(2 x 10 =**

5. State and prove multiplication theorem of probability for independent events.  
 6. A coffee connoisseur claims that he can distinguish between a cup of instant coffee and a cup of percolator coffee 75% of the time. It is agreed that his claim is accepted if he correctly identifies at least 5 of the 6 cups. Find his chances of having the claim (i) accepted, (ii) rejected, when he does have the ability he claims.  
 7. Prove that exponential distribution has a lack of memory property.  
 8. If particles are emitted from a radioactive source a rate of 20 per hour, find the probability that

exactly 5 particles are emitted during a 15 minutes period.

**SECTION C – K4 (CO3)**

**Answer any TWO of the following** (2 x 10 = 20)

9. State and prove Bayes' theorem.
10. (a) A problem in statistics is given to 3 students, whose chance of solving it are  $\frac{1}{2}$ ,  $\frac{3}{4}$  and  $\frac{1}{4}$  respectively. What is the probability that the problem will be solved if all of them try independently?  
(b) Given  $P(A) = 0.4$ ,  $P(A \cup B) = 0.7$ . Find  $P(B)$ , if A and B are mutually exclusive. (6+4)
11. Derive the first three moments of Poisson distribution.
12. If the initial stage probability distribution of a Markov chain is  $P^{(0)} = \begin{pmatrix} 5/6 & 1/6 \end{pmatrix}$  and the tpm of the chain is  $\begin{pmatrix} 0 & 1 \\ 1/2 & 1/2 \end{pmatrix}$ . Find the probability distribution of the chain after 2 steps.

**SECTION D – K5 (CO4)**

**Answer any ONE of the following** (1 x 20 = 20)

13. The daily consumption of milk in a city, in excess of 20,000 litres, is approximately distributed as a gamma variate with parameters  $a = 1/10,000$  and  $\lambda = 2$ . The city has a daily stock of 30,000 litres. What is the probability that the stock is insufficient on a particular day?
14. (a) Derive mean and variance of Geometric distribution.  
(b) Find the probability density of geometric distribution if the value of p is 0.42;  $x = 1, 2, 3$  and also calculate the mean and variance. (10+10)

**SECTION E – K6 (CO5)**

**Answer any ONE of the following** (1 x 20 = 20)

15. Derive the moments of normal distribution.
16. Suppose the customers arrive at a bank according to Poisson process with a mean rate of 3 per minute. Find the probability that during a time interval of 2 mins (i) exactly 4 customers arrive (ii) more than 4 customers arrive and (iii) fewer than 4 customers arrive.

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